Straight Line Motion - Homework

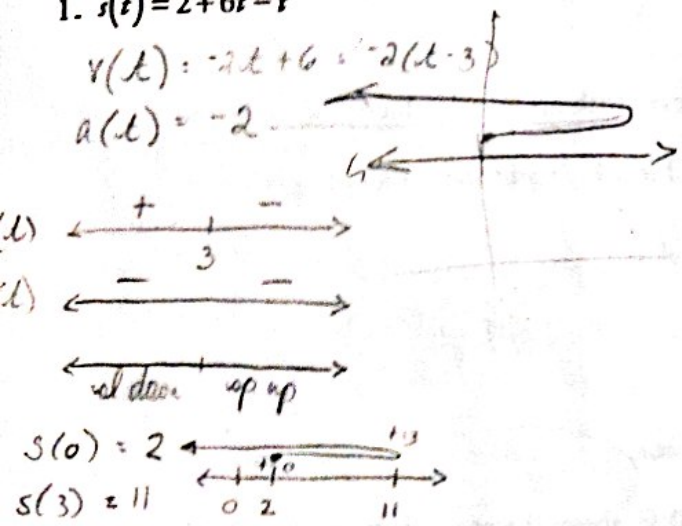
A particle is moving along a horizontal line with position function as given. Do an analysis of the particle's direction, acceleration, motion (speeding up or slowing down), and position.

1. $s(t) = 2 + 6t - t^2$

$-t^2 + 6t + 2$

$v(t) = -2t + 6 = -2(t-3)$

$a(t) = -2$



$s(0) = 2$

$s(3) = 11$

2. $s(t) = t^3 - 6t^2 + 9t - 4$

$v(t) = 3t^2 - 12t + 9 = 3(t^2 - 4t + 3)$

$3(t-3)(t-1)$

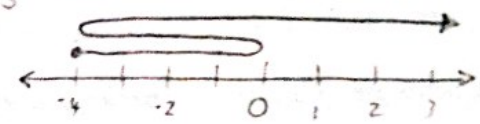
$a(t) = 6t - 12$

$6(t-2)$

$s(0) = -4$

$s(1) = 0$

$s(3) = -4$



3. $s(t) = t^3 - 9t^2 + 24t - 1$

$v(t) = 3t^2 - 18t + 24$

$3(t^2 - 6t + 8)$

$3(t-4)(t-2)$

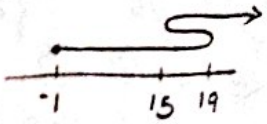
$s(0) = -1$

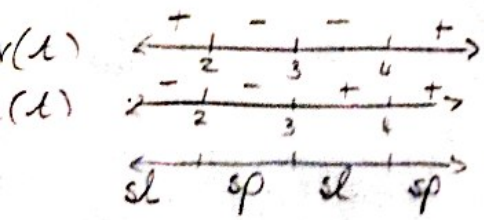
$s(2) = 19$

$s(4) = 15$

$a(t) = 6t - 18$

$6(t-3)$





4. $s(t) = t + \frac{9}{t+1} + 1$

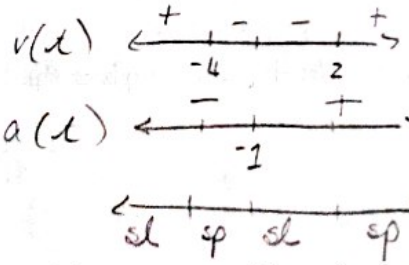
$(t+4)(t-2)$

$v(t) = 1 - \frac{9}{(t+1)^2} = \frac{t^2 + 2t - 8}{(t+1)^2}$

$a(t) = \frac{9 \cdot 2(t+1)}{(t+1)^4} = \frac{18(t+1)}{(t+1)^4}$

$s(0) = 10$

$s(2) = 6$



5. A 45-caliber bullet fired straight up from the surface of the moon would reach a height of $s = 832t - 2.6t^2$ feet after t seconds. On Earth, in the absence of air, its height would be $s = 832t - 16t^2$ feet after t seconds. How long would it take the bullet to hit the ground in either case?

Earth

$s = 832t - 16t^2 = 0$

$s = 16t(52 - t) = 0$

$t = 0$ or $t = 52$ s

Moon

$s = 832t - 2.6t^2 = 0$

$s = 2.6t(320 - t) = 0$

$t = 0$ or $t = 320$ s

6. A ball fired downward from a height of 112 feet hits the ground in 2 seconds. Find its initial velocity.

$S = S_0 + V_0 t - \frac{1}{2} g t^2$

$0 = 112 + V_0(2) - 16(2)^2$

$V_0 = -24$ ft/s

7. A projectile is fired vertically upward (earth) from ground level with an initial velocity of 16 ft/sec.

a. How long will it take for the projectile to hit the ground?

$S = 0 + 16t - 16t^2 = 0$

$-16t(t-1) = 0$

$t = 0$ $\quad t = 1$ sec

b. How high will the projectile get?

$V = -32t + 16 = 0$

$-16(2t-1) = 0$

$t = 1/2$

$S = 0 + 16(1/2) - 16(1/2)^2 = 4$ ft

8. A helicopter pilot drops a package when the helicopter is 200 ft. above the ground, rising at 20 ft/sec.

a. How long will it take for the package to hit the ground?

$S = 200 + 20t - 16t^2$

Quad. Form

$t = -2.966$ $\quad 4.215$ sec

b. What is the speed of the package at impact?

$V = 20 - 32(4.215)$

$V = -114.88$ ft/sec

9. A man drops a quarter from a bridge. How high is the bridge if the quarter hits the water 4 seconds later?

$0 = S_0 + 0(4) - 16 \cdot 4^2$

$S_0 = 256$ ft

10. A projectile fired upward from ground level is to reach a maximum height of 1,600 feet. What is its initial velocity?

$1600 = 0 + V_0 t - 16t^2$

$1600 = 32t \cdot t - 16t^2$

$1600 = 16t^2$

$t^2 = 100$

$t = 10$ sec

$V = V_0 - gt$

$0 = V_0 - 32t$ at max height

$V_0 = 32t$

$V_0 = 32(10)$

$V_0 = 320$ ft/sec

11. A projectile is fired vertically upward with an initial velocity of 96 ft/sec from a tower 256 feet high.

a. How long will it take for the projectile to reach its maximum height.

$v = 96 - 32t = 0$

$t = 3$ sec

b. What is its maximum height?

$s = 256 + 96(3) - 16(3)^2$

$s = 400$ ft

c. How long will it take the projectile to reach its starting point on the way down?

$256 = 256 + 96t - 16t^2$

$t = 6$ sec

d. What is the velocity when it passes the starting point on the way down?

$v = 96 - 32(6)$

$v = -96$ ft/sec

f. How long will it take to hit the ground?

$s = 256 + 96t - 16t^2$

$s = -16(t-8)(t+2)$

$t = 8$ sec $t = -2$

e. What will be its speed when it impacts the ground?

$v = 96 - 32(8) = -160$

speed = 160 ft/sec

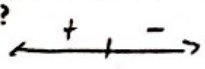
12. Steve's car runs out of gas as it goies up a hill. The car rolls to a stop then starts rolling backwards. As it rolls, its displacement $d(t)$ from the bottom of the hill at t seconds since the car ran out of gas is given by:

$d(t) = 125 + 31t - t^2$.

a. When is his velocity positive? What does this mean in real world terms?

$v = -2t + 31$

$t = 15\frac{1}{2}$



$0 \le t \le 15\frac{1}{2}$

b. When did the car start to roll backwards? How far was it from the bottom of the hill at that time?

$15\frac{1}{2}$ sec

$d(15\frac{1}{2}) = 125 + 31(15\frac{1}{2}) - (15\frac{1}{2})^2 = 365.25$ ft

c. If John keeps his foot off the brake, when will he be at the bottom of the hill?

$125 + 31t - t^2 = 0$

Q.F.

~~-3.6~~ or 34.6 sec

d. How far was John from the bottom of the hill when he ran out of gas?

$d(0) = 125$ ft